Combinatorial Games on Graphs (Dr. Robert Bell, research mentor)

Description: Students will explore two player, complete information games, such as COPS AND ROBBER [3], that are played on a combinatorial graph. In COPS AND ROBBER, the first player controls a fixed number of cops or police. You can imagine these as represented by pawns on a board, the space of the board corresponding to vertices of some combinatorial graph and the adjacent spaces corresponding to adjacent vertices, i.e., vertices that are joined by an edge. Suppose the first player has k cop pawns. On the first turn, she assign each cop to a vertex of G. The second player controls a single pawn called the robber. After the cops are placed, he assigns the robber to a vertex. Thereafter, players alternate turns and move each of their pawns to an adjacent vertices on their turn. The cops win if the robber is captured; the robber wins if this outcome can be avoided. The fundamental problem is to determine the cop number of G: the least k such that the cops have a winning strategy, i.e., a strategy which will result in a win for the cops no matter what strategy is chosen by the robber.

One can modify the rules, and this can lead to interesting variations. One extreme is that the robber has to remain stationary and the cops cannot see where the robber is located; this is a well-known problem of computer science, namely, the search problem. Another variation might restrict the information given to the cops, e.g., perhaps the cops only know the location of the robber on every other turn. Generally, whether or not a variation is suitable for further research depends on whether or not it leads to "good" mathematics, e.g., if it connects to some topic in graph theory or combinatorial game theory that is known to have a mathematically interesting theory.

This project is very suitable for a summer research project since one can immediately work with examples, formulate conjectures, test ideas, and read (relatively accessible) research papers. No prior experience with graph theory is required. Students who want to work on this project do need to have some experience with writing proofs, e.g., a first course in which one learns how to use mathematical induction and argue by contradiction. An interest in or experience with computer programming is desirable. I especially enjoy thinking about graphs which arise from problems in abstract algebra, geometry, or topology; so, you'll almost certainly learn about some connections to these areas of mathematics during our investigations if we work together this summer. Please reach out via e-mail (`bellro@msu.edu`)if you have questions about this project.

References

[1] Aigner, M. and Fromme, M., A game of cops and robbers, Discrete Appl. Math., 8 (1984), pp. 1–11.

[2] Bonato,A. *An invitation to pursuit-evasion games and graph theory*, Student Mathematical Library, 97, AMS, Providence, RI, 2022.

[3] Bonato, A.; Nowakowski, R. J.; *The game of cops and robbers on graphs*. Student Matheamtical Library, 61. AMS, Providence, RI, 2011. ISBN:978-0-8218-5347-4.