Project Description
The Scattering Transform on Graphs

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1 Introduction

The *scattering transform* is a wavelet-based deep learning architecture with a structure modeled after a Convolutional Neural Network (CNN). Similar to a standard CNN, the scattering transform is a multilayered network which at each layer first convolves an inputted function $f \in \mathbf{L}^2(\mathbb{R}^n)$, referred to as *the signal*, with another function called *the filter*, and then applies a non-linearity, referred to as *the activation function*. Standard CNNs typically use the activation function $ReLu(x) = \max\{x, 0\}$ and solve a highly non-convex optimization problem to determine the best possible choice of filters based off of training data. The scattering transform, on the other hand, uses the activation function $M(x) = |x|$ and, more importantly, uses predetermined wavelet filters rather than filters obtained by solving an optimization problem. These modifications lead to a network that can be analyzed with rigorous mathematics and which provably possesses desirable properties. Moreover, in addition to these theoretical properties, the scattering transform is well-suited to limited labeled-data environments since it does not to learn its filters from training data.

Both traditional CNNs, and also the original scattering transform as introduced in [7], are designed to classify data with a Euclidean structure. However, many data sets of interest, including social networks, regulatory networks in genetics, and surfaces in computer graphics, have an intrinsically non-Euclidean structure and are better modeled as manifolds or graphs. This has lead to the new field of geometric deep learning which aims to transfer deep learning methods to these non-Euclidean environments (see e.g. [1]). In particular, various graph-based formulations of the scattering transform have been recently introduced by Gao et al. [5], Gama et al. [4], and Zou et al, [9]. These networks have been be applied to a variety of graph learning tasks, including authorship attribution, the classification of social networks, and community detection.

Our proposed project will be based upon the graph scattering transform in [5], which uses wavelets constructed in from a lazy graph random walk at different time scales. Both random walks on graphs and the aforementioned graph learning tasks are relatively intuitive topics. We, therefore, believe this will provide an excellent introduction to recently developed mathematics for students who do not necessarily have much formal mathematical training.

The REU will begin by providing students the necessary background on graph random walks, the scattering transform, and convolutional neural networks. We will then teach them how to implement the graph scattering transform as formulated in [5]. Once the students are comfortable implementing the graph scattering transform, they will conduct numerical experiments through applications to network structured data that arises in the study of complex social, economic, and ecological systems. This may include networks underlying Agent Based Models [3] as well as directed graphs such as Fuzzy Cognitive Maps, as developed by Kosko in [6], and Causal Loop Diagrams as employed by [2] and [8]. These networks represent the magnitude and direction of causal relationships among elements of a system. Tasks of interest can include the clustering of network typologies, community detection, or the structural analysis of a complex system.

References

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