Embedded Graph Smoothing

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I. Collaborators

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II. Basic idea

At this point, it is cliche to point out that data science intersects all aspects of our lives, online or not. While often this is interpreted as a need to build analysis methods for *big* data, we also need methods for analyzing *complex* data. Where the former is concerned with cases where we have terabytes upon terabytes of input, the latter is focused on finding and quantifying interesting structure in data. The field of topological data analysis (TDA) [3] takes the second vantage point by providing quantifiable, comparable, robust, and concise summaries of the shape of data. This may mean measuring shape in an obvious shape-focused context such as quantifying shape and structure in plant leaves, or could mean understanding high-dimensional point clouds by providing lower dimensional skeletons of the underlying embedded structures.

In this research project, we will focus on graphical signatures of data, specifically embedded graphs. A graph is a combinatorial object consisting of vertices and edges, however we can also view it as a continuous topological space, particularly when we are concerned with drawings of the graph. That is, a drawing of a graph G is a continuous map $\Phi : G \to \mathbb{R}^2$.

Our goal is to find methods to compare two graph embeddings [2]. This is particularly useful in applications such as comparison of geographic networks such as rivers or roads. We are particularly interested in creating quality measures for map reconstructions where, for example, we have a reconstructed road network given GPS trajectory data. The main idea is to provide quality guarantees for computed approximations of a ground truth map.

We will utilize the idea of smoothing a graph, which arose in the context of defining a metric for Reeb graphs [4]. The Reeb graph originated in the context of Morse theory where it provides a graph given as input a (well-behaved) topological space with a (well-behaved) real valued function by encoding information about connected components of level sets of the function. We simply work with these objects as being a graph with an $\mathbb{R}$ valued function, $f : G \to \mathbb{R}$. Then the smoothing construction provides a 1-parameter family of Reeb graphs which serves to essentially simplify the graph and function by collapsing small loops and shortening tails.

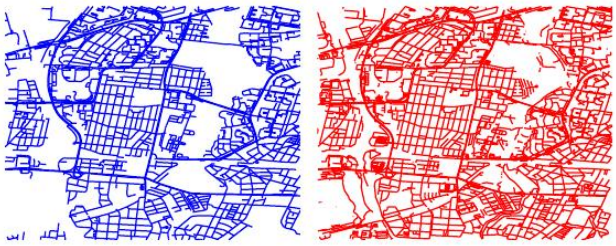

Fig. 1. Two reconstructed maps of Berlin. How do we compare the two graphs? From [1].

By picking an orientation in the plane, $\omega \in S^1$, any drawing of a graph $\Phi : G \to \mathbb{R}^2$ can provide a single valued function for the graph $\Phi_\omega : G \to \mathbb{R}$ by looking at the height of the graph in that particular direction. We will investigate how smoothing of this resulting Reeb graph can be used for comparison of two graph drawings, $\Phi : G \to \mathbb{R}^2$ and $\Psi : H \to \mathbb{R}^2$.

III. Summer research goals

- Understand the Reeb graph smoothing construction.
- Compute smoothings for example graphs, by hand and/or by computer.
- Develop a comparison measure for two embedded graphs using this technique.
- Apply the results to GIS data.

IV. Useful background knowledge

Note that while experience with any of these directions will be useful, they are not required as we can pick up what is needed as we go along.

- Graph theory/discrete math.
- Point-set topology.
- Some experience with python for computation.

References

[1] Mahmuda Ahmed, Brittany Terese Fasy, and Carola Wenk. “Local persistent homology based distance between maps”. In: *Proceedings of the 22nd ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems*. ACM. 2014, pp. 43–52.

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[3] Elizabeth Munch. “A User’s Guide to Topological Data Analysis”. In: *Journal of Learning Analytics* 4.2 (2017). DOI: 10.18608/jla.2017.42.6.

[4] Vin de Silva, Elizabeth Munch, and Amit Patel. “Categorified Reeb Graphs”. In: *Discrete & Computational Geometry* (2016), pp. 1–53. ISSN: 1432-0444. DOI: 10.1007/s00454-016-9763-9.