Computing and experimenting with Hecke algebras

The goals of this project are twofold. First, to write computer programs, using the compute algebra software SAGE, to compute *localizations of Hecke algebras*. Second, run these programs with various inputs and analyze the outputs. The second part is the most fun – you get to look for patterns in the data, formulate conjectures about what the pattern is, and then test your conjecture experimentally by doing more computations.

From the point of view of this project, Hecke algebras are simply certain finite collections of square matrices that all commute with each other. Understanding where these matrices come from requires much more advanced mathematics, like number theory, algebraic geometry, and algebraic topology. But luckily, for this project, we don't need to know any of that – SAGE is able to compute the matrices for us. What we want to do is take these matrices as an input, and produce as a output a presentation of the algebra in terms of polynomials, which will be more familiar, easier to understand, and easier to do algebra operations on.

During this project you will:

- Learn compute programing in SAGE (which uses the Python language),
- Practice and improve linear algebra skills,
- Learn abstract algebra from a computational perspective,
- Perform experiments in number theory.

The software SAGE is open source. Any completed programs can be submitted to SAGE for peer-review, and can be published and included in future versions of SAGE.

To learn more about SAGE, go to [Sag19]. There, you can access the SAGE software for free (there is even a browser-based interface using CoCalc – not download required). You are encouraged to try out the tutorials offered there. To see examples of these types of computations that can be done, look at the tables and Examples sections in the introductions of the papers [WWE20] and [WWE18].

References

[Sag19] Sage Developers. *SageMath, the Sage Mathematics Software System*, 2019. `https://www.sagemath.org`.

[WWE18] Preston Wake and Carl Wang-Erickson. The Eisenstein ideal with squarefree level. Preprint, https://arxiv.org/abs/1804.06400, 2018.

[WWE20] Preston Wake and Carl Wang-Erickson. The rank of Mazur's Eisenstein ideal. *Duke Math. J.*, 169(1):31–115, 2020.