GUESSING NUMBERS AGAINST LIARS

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A well-known number guessing game is set up thus:

> I think of a integer between 1 and 100. You are to try and guess it. To help you along: each time you guess incorrectly I will tell you whether my secret number is bigger or smaller than your guess. How many tries will you need to guarantee a win?

The answer to this is 7, and the procedure is that of a binary search: each time you try to eliminate at least half of the (remaining) numbers from contention using the clue given.

More generally the game can be formulated in the following way:

> Given a finite set S of k elements, and a set $\mathscr{S}$ of subsets of S (the "legal guesses"). I select an element $s_0 \in S$. At each turn, you are allowed to select a subset $A \in \mathscr{S}$ and ask whether $s_0 \in A$. Can you design an algorithm that recovers s_0 using as few queries as possible?

In the familiar game, the subset $\mathscr{S}$ of legal guesses are given by

$$\Big\{\{1\}, \{1,2\}, \{1,2,3\}, \ldots, \{1,2,\ldots,99\}\Big\}.$$

For the more general problem, an algorithm can always be found if every singleton set $\{s_0\}$ can be written as the intersection of sets which either belongs to $\mathscr{S}$ or whose complement belongs to $\mathscr{S}$.

Questions of this type belong in *information theory*: it asks about efficient *encoding* of information into a sequence of binary (true or false) responses. In real-life scenario, however, the transmission of information and the encoding of information may be error prone. This can come from systematic errors of the instruments involved, or just from random noise that may be injected into the results. We would therefore like to build systems and algorithms that have at least some degree of *fault tolerance*.

One way to model the injection of errors into the process is to allow the *host* in the number guessing game to *lie*. Questions of these type were first considered by Alfréd Rényi in 1961, and over the past 60 years a lot has been discovered about such games, based on exactly "how much lying" the host is allowed to do.

In this project we will start by learning about what is already known about these types of problems. After this we will try to chip away at the unknown by seeing either we can beat some of the best known algorithms for a previously studied game, or studying a version that you come up with yourself.

Further Readings.

- Ingrid Daubechies, C. Sinan Güntürk, Yang Wang, and Özgür Yılmaz, "The Golden Ratio Encoder", IEEE Transactions on Information Theory (56), 2010.
- Andrzej Pelc, "Searching games with errors–fifty years of coping with liars". Theoretical Computer Science (270), 2002.