Random Matrix Product States

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Quantum mechanics concerns itself with the calculation of coefficients associated with a *wave function* ψ that encode probabilities about the system. For example, the Stern-Gerlach experiment showed that electrons interact with magnetic fields in a rotation-invariant way called *spin* [7]. An example of the wave function of an electron's spin could look like

$$\psi = \frac{1}{\sqrt{2}} \left(| \uparrow \rangle + | \downarrow \rangle \right)$$

meaning the spin is pointing up or down with a probability of 50% in either case. When there are many particles, however, this calculation becomes exceedingly difficult due to the exponential increase in the number of degrees of freedom of the system. For example, just keeping track of the spins of 10 electrons requires one to perform $2^{10} = 1024$ calculations.

Enter the theory of *matrix product states* (MPS) which was pioneered in the papers [2, 6, 8] introduced a new way to calculate these probabilities with far greater efficiency. Namely, one need only specify a family of complex matrices $A_1, A_2, \ldots, A_n$ and calculate a desired basis state ϕ. Then if ψ is an MPS described by the A_j's then the probability of finding ψ in ϕ is

$$\text{Prob} = |\operatorname{Tr}(A_1 \cdot \cdots \cdot A_n)|^2$$

At first this doesn't look that efficient, but the key insight is that the dimension of the matrices can be smaller than the dimension of the underlying state space. Using our previous example, the number of computations for a matrix product state of 10 particles could be as low as ≈ 20. Not only this, but MPS are actually quite good at approximating arbitrary states. The theory of MPS (and their older siblings, the *tensor networks*[1]) has become a cornerstone of modern many-body quantum systems.

Recent interest has turned toward *disordered* MPS as a means to understand quantum systems with on-site disorder [4, 3, 5]. I plan to have our group investigate various properties of these new disordered matrix product states using software methods. One such property is the *correlation decay*, which tells us something about how entangled a given quantum system is at a given distance.

Knowledge of undergraduate quantum mechanics, and coding experience is not required but would be helpful. Other prerequisites are linear algebra or Calc 3, some course that used proofs, and enthusiasm :)

Further Reading

- *Matrix Analysis*. Roger A. Horn and Charles R. Johnson. (2nd ed). Section 7.3.
- *Matrix Product States: A Variational Approach to Strongly Correlated Systems*. Andreas Haller. Bachelor Thesis. University Mainz. Chapters 1-2.

References

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