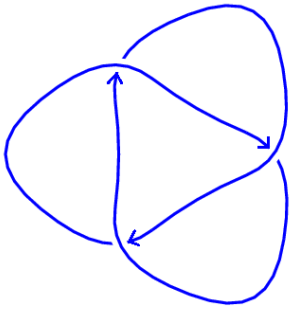
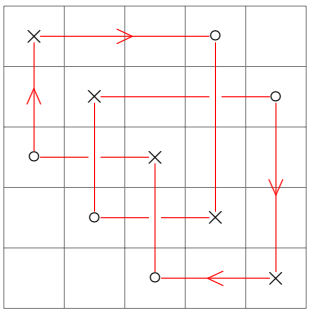
Combinatorial knot theory (Gerhardt and Hedden). Knot theory provides a mathematical framework in which to study the intuitive subject of tangling up pieces of string in space. This project will introduce students to knot theory and its interesting interactions with combinatorics. Much interest has recently been generated by a combinatorial device, known as a *grid diagram*, for the codification and analysis of knots. A grid diagram is a diagram of a knot which has the property that it is a union of horizontal and vertical line segments with the rule that vertical line segments always pass over horizontal segments. One of the key features of grid diagrams is that they allow for the encoding of a knot with a minimal amount of data. Indeed, the only important feature about a grid diagram is the relative horizontal and vertical positions of the endpoints of the line segments comprising it. For example, below we show a knot projection and a grid diagram for the trefoil knot.





Grid diagrams have appeared in various guises but have gained a considerable amount of attention over the past twenty five years, notably in the work Cromwell [1], Dynnikov [2], and Manolescu, Ozsváth, and Sarkar [4].

In this research project students will study combinatorial aspects of knot theory using grid diagrams. Avenues for investigation include understanding the minimal number of vertical and horizontal segments for various families of knots (the so-called "grid number" of knots), exploring colorability properties of knots through grid diagrams, exploring polynomial knot invariants through grid diagrams and permutations, and many more. These questions are combinatorial in nature and accessible to students without prior background in topology.

References

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[2] I. A. Dynnikov, *Arc-presentations of links: monotonic simplification*, Fund. Math. **190** (2006), 29–76. MR 2232855 (2007e:57006)

[3] Manolescu, C.; Ozsváth, P.; Sarkar, S., *A combinatorial description of knot Floer homology*, Ann. of Math. (2) **169** (2009), 633–660.

[4] Manolescu, C.; Ozsváth, P.; Sarkar, S., *A combinatorial description of knot Floer homology*, Ann. of Math. (2) **169** (2009), 633–660.