Combinatorial Games on Graphs (Dr. Robert Bell, research mentor)

Description: Students study the two player, complete information game COPS AND ROBBERS [2] played on a combinatorial graph G. One player controls a fixed number k of *cops* and assigns each to a vertex of G. Another player controls a *robber*: after the cops are placed, the robber is assigned to a vertex. Players alternate turns and move any number of their pawns to adjacent vertices on a turn. Cops win if the robber is captured; robber wins if this outcome can be avoided. The fundamental problem is to determine the cop number of G: the least k so that the cops can always win. Changing the rules leads to interesting variations. One extreme is the search problem in computer science where the cop can only see adjacent vertices and the robber is immobile.

Students can immediately work with examples, formulate conjectures, test ideas, and read papers. Results from the 2014 REU [1] suggested the following variation: the cops *weakly win* if the robber is prevented from visiting any vertex infinitely often [4]. In subsequent years, students have studied the weak cop number of products of graphs and some families of planar graphs [3]. Projects for this REU:

1. Statements about the cop number of finite graphs sometimes remain true for the weak cop number of an infinite graph. Which statements hold? What additional hypotheses are sufficient for a correct statement? Methods of the previous REUs often apply to give upper bounds. New methods and computer programs are needed to establish lower bounds.
2. Containment is a variant of cops and robbers in which cops occupy edges and move to incident edges so as to surround a robber who occupies vertices and moves to adjacent vertices [2]. Investigate the game of "weak containment" where cops try to prevent the robber from visiting any vertex infinitely often.

References

[1] Ball, Taylor; Bell, Robert W.; Guzman, Jonathan; Hanson-Colvin, Madeleine; Schonsheck, Nikolas. *On the cop number of generalized Petersen graphs*, Discrete Math. 340 (2017), no. 6, 1381–1388.

[2] Bonato, A.; Nowakowski, R. J.; *The game of cops and robbers on graphs*. Student Matheamtical Library, 61. AMS, Providence, RI, 2011. ISBN:978-0-8218-5347-4.

[3] Jordan DuBeau and Beth Matys, *Cops and robbers on infinite graphs*, 2015. (Final technical report from the SURIEM 2015 REU at MSU.)

[4] Lehner, F., *Cops, robbers, and infinite graphs*, arXiv:1410.8412v4 [math.CO].