# BROWNIAN MOTION AND RANDOM FRACTALS

YIMIN XIAO (MICHIGAN STATE UNIVERSITY)

Brownian motion is not only a beautiful and fascinating object in mathematics, but also a powerful stochastic model that has been applied in many scientific fields, from physics to finance, to biological sciences. The same can be said about fractal geometry, whose birth was due in great measure to the work of Benoit Mandelbrot (1982). Both topics have brought many new ideas and new methods into mathematics and statistics, and continue to find new applications. It is therefore very important for students to learn these exciting topics.

Since Brownian motion is not differentiable and generates various interesting fractal sets and measures, it is natural (and necessary) to apply tools from Fractal Geometry (e.g. Hausdorff dimension, packing dimension) to study fine properties of Brownian motion. Since the pioneering work of Lévy (1953) and Taylor (1953), there has been an enormous literature on sample path properties of Brownian motion. We refer to Taylor (1986), Xiao (2004), Mörters and Peres (2010), and Khoshnevisan and Xiao (2015) for further information. More recently, macro-scale fractal properties of Brownian motion and stochastic partial differential equations have been studied by Khoshnevisan and Xiao (2017), Khoshnevisan, Kim and Xiao (2017, 2018).

The objective of this research group is to investigate micro-scale and macro-scale fractal properties of Brownian motion and other related stochastic processes. The following research project is an example.

Let $B = \{B(t) : t \geq 0\}$ be Brownian motion with values in $\mathbf{R}^d$, let $\mathrm{Gr}B(\mathbf{R}) = \{(t; B(t)) : t \geq 0\}$ be its graph set, and let

$$M_k = \{x \in \mathbf{R}^d : \exists \text{ distinct } \ t_1, \ldots, t_k \text{ such that } \ B(t_1) = \cdots = B(t_k) = x\}$$

be the set of $k$-multiple points. Determine the macro-scale Hausdorff, Minkowski, and packing dimensions of $\mathrm{Gr}B(\mathbf{R})$ and $M_k$.